GroupHomework #1

Death and Taxes

A U.S. taxpayer pays no federal tax on the first $15,000 of annual income, 15% on the next $25,000 of income and 28% on the all earnings above $40,000. Let x denote a taxpayer's income (in dollars) and let *T(x)* denote the tax owed (in dollars) at that income level.
a) Compute *T(10000)*, *T(20000)*, *T(30000)*, *T(40000)*, and *T(50000)*.
b) Give a piecewise defined formula for *T(x)* and draw the graph of *T(x)* for $0 \le x \le 60000$. Verify that when you use this formula for the values 10000, 20000, 30000, 40000 and 50000 your answers match your answers from part (a).
c) If a taxpayer earning $56,789 earned one more dollar of income, the tax owed would increase by 28 cents. If a taxpayer earning $34,567 earned one more dollar of income, the tax owed would increase by 15 cents. The first taxpayer has a **marginal tax rate** of 28%, while the second taxpayer has a **marginal tax rate** of 15%. Let *MR(x)* denote the marginal tax rate (as a decimal) at an income level of x. Give a piecewise defined formula for *MR(x)*.
d) Draw the graph of *MR(x)* for $0 \le x \le 60000$. Describe the relation between *T(x)* and *MR(x)* in your own words using complete sentences.

Spherical Tank

A spherical tank has a diameter of 10 feet. Water is flowing into the tank at a constant rate of 4 cubic feet per minute.
a) How long does it take to fill the tank of water?
Let *D(t)* be the depth of water in the tank t minutes after water begins flowing into the tank.
b) What is the Range of values for *D(t)*?
c) Is the graph of *D(t)* increasing or decreasing? Explain?
d) Where is the graph of *D(t)* concave up and where is the graph concave down? Explain why this happens?

Heaviside Function

The Heaviside function is defined as $H(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$ is 0 for negative values and 1 for positive values. It frequently is used in engineering to represent a sudden change.

a) Plot the Heaviside function for a suitable range of t.
b) Find a function $F_1(t)$ of the form $F_1(t) = A \times H(t + c)$, which is 0 for $t < 3$ and 5 for $t > 3$. You will need to find the appropriate values for A and c.
c) Find a function $F_2(t)$ of the form $F_2(t) = B + A \times H(t + c)$, which is 2 for $t < 1$ and 1 for $t > 1$. You will need to find the appropriate values for A, B and c.

d) Find a function $F_3(t)$ of the form $F_3(t) = f(t) + g(t) \times H(t + c)$, which is $(t - 2)^2$ for $t < 2$ and $2 - t$ for $t > 2$. You will need to find the appropriate value for c and appropriate functions f(t) and g(t).

Why use e?

A function which grows exponentially is generally written in the form $f(t) = y_0 e^{kt}$. This question is to show that an exponential function can always be written in this form.

a) Find values of y_0 and k so that $2^t = y_0 e^{kt}$.

b) Find values of y_0 and k so that $3^{4t} = y_0 e^{kt}$.

c) Find values of y_0 and k so that $5^{t+2} = y_0 e^{kt}$.

d) Find values of y_0 and k so that $A \times B^{Ct+D} = y_0 e^{kt}$. These values will depend on A, B, C and D