

Group Homework #3

Estimate

a) Fill in the following table with values of $y = e^x$, for $x=0.1$, $x=0.01$, $x=0.001$, $x=-0.001$, $x=-0.01$, and $x=-0.1$.

x	e^x
-0.1	
-0.01	
-0.001	
0.001	
0.01	
0.1	

Use these to estimate the value of $\frac{dy}{dx}$ at $x=0$.

b) Fill in the following table with values of $y = 5^x$, for $x=0.1$, $x=0.01$, $x=0.001$, $x=-0.001$, $x=-0.01$, and $x=-0.1$.

x	5^x
-0.1	
-0.01	
-0.001	
0.001	
0.01	
0.1	

Use these to estimate the value of $\frac{dy}{dx}$ at $x=0$.

c) Fill in the following table with values of $y = 10^x$, for $x=0.1$, $x=0.01$, $x=0.001$, $x=-0.001$, $x=-0.01$, and $x=-0.1$.

x	10^x
-0.1	
-0.01	
-0.001	
0.001	
0.01	
0.1	

Use these to estimate the value of $\frac{dy}{dx}$ at $x=0$.

d) How are the numbers $\ln(e)$, $\ln(5)$ and $\ln(10)$ related to your answers in parts (a)-(c).

Is it possible?

Suppose that $f(x)$ is a continuous and differentiable function with a local maximum at 2 and an inflection point at 3. Consider each question separately.

- Is it possible that $f(2)=1$? If not, explain why not. Justify your answer.
- Is it possible that $f'(2)=1$? If not, explain why not. Justify your answer.
- Is it possible that $f''(2)=1$? If not, explain why not. Justify your answer.
- Is it possible that $f(3)=1$? If not, explain why not. Justify your answer.
- Is it possible that $f'(3)=1$? If not, explain why not. Justify your answer.
- Is it possible that $f''(3)=1$? If not, explain why not. Justify your answer.

Threshold population

The rate at which a population increases or decreases depends on the size of the population (there are more births in a large city than in a small town). Suppose that the population of fish in a lake follows the differential equation: $\frac{dP}{dt} = 0.05P - 65$, where P

is the population of fish and t is time.

- Suppose the population of fish is 200. Is the population increasing or decreasing? Explain.
- Suppose that population of fish is 2000. Is the population increasing or decreasing? Explain.

Many species have a threshold population, written as P_T , below which the population will decrease and eventually die out and above which the population will increase.

- Determine the threshold population for this population of fish.

Bounds

Suppose that $C(1)=-2$ and $C'(t) \leq 3$ for all values of t .

- Show that $C(2) \leq 1$.
- Show that $C(5) \leq 10$.
- Is $C(9) < 31$? Justify your answer.
- Is $C(-5) > -23$? Justify your answer.
- Could $C(4)=8$? Justify your answer.