Exam 1 Solutions

1) $C(10) = 23{,}000$, $-30 \le C'(t) \le -15$. Find a and b so that $a \le C(25) \le b$.

The speed limit law says that if

$f'(x) \le M$ then

$$(f(b) - f(a)) \le M(b-a)$$

So, with $a = 10$, $b = 25$ and $M = -15$,

$$C(25) - C(10) \le -15(25-10)$$

$$\Rightarrow \quad C(25) \le 23{,}000 - 225 = 22{,}775$$

And vice-versa

$$C(25) - C(10) \ge -30(25-10)$$

$$\Rightarrow \quad C(25) \ge 23{,}000 - 450 = 22{,}550$$

So $\quad 22775 \ge C(25) \ge 22{,}550$

2) $f'(x) = -13$, $f(23) = -18$, find $f(x)$

f must be a linear function with slope -13, $f(x) = -13x + b$. To find b,

$$f(23) = -13(23) + b = -18$$
$$\Rightarrow b = 281$$

$$\boxed{f(x) = -13x + 281}$$

3) The dotted line is f
light line is f'
dark line is f''

Where f has stationary points, f' has roots and where f' is positive (negative) f is increasing (decreasing).

Where f' has stationary points f'' has roots and where f'' is positive (negative) f is concave up (down.)

4) $h(t) = -16t^2 + 38t + 74$, $v(t) = -32t + 38$

a) $h(0) = -16(0)^2 + 38(0) + 74 = \boxed{74 \text{ feet}}$

b) $v(0) = -32(0) + 38 = \boxed{38 \text{ feet/sec.}}$

Thrown up because $v(0) > 0$.

c) Maximum height when $v(t) = 0$

$-32t + 38 = 0 \Rightarrow t = \frac{38}{32} = \boxed{\frac{19}{16} \text{ seconds}}$

$h\left(\frac{19}{16}\right) = -16\left(\frac{19}{16}\right)^2 + 38\left(\frac{19}{16}\right) + 74$

$\cong \boxed{96.6 \text{ feet}}$

d) The ball landed when $h(t) = 0$.

$-16t^2 + 38t + 74 = 0$

$\Rightarrow \quad t = \frac{-38 \pm \sqrt{38^2 + 4(16)74}}{-32} \cong \boxed{3.64 \text{ seconds}}$

5) a) increasing: $(-\infty,-3)\cup(0,\infty)$ because $f'>0$

decreasing: $(-3,0)$ because $f'<0$

b) $f'(x)=0$ at $x=-3,0,2$

c) $x=-3$ is a local max. by the first derivative test

$x=0$ is a local min. by the first derivative test

d) concave up: $(-2,.5)\cup(2,\infty)$ because f' is increasing

concave down: $(-\infty,-2)\cup(.5,2)$ because f' is decreasing

e) inflection pts: $x=-2,.5,2$

because the concavity changes there.

6) a) $$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

or

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

b) $$f'(4) = \lim_{h \to 0} \frac{2\sqrt{4+h} - 1 - (2\sqrt{4} - 1)}{h}$$

c) $$f'(4) \cong \frac{f(3.99) - f(4)}{-.01} = \frac{2.995 - 3}{-.01} = .5$$

$$\cong \frac{f(4.01) - f(4)}{.01} = \frac{3.005 - 3}{.01} = .5$$

So $f'(4) = .5$