Exam 2 Solutions

1) a) $f(x) = 1/x$

$$f'(x) = -x^{-2} = -\frac{1}{x^2}$$

b) $f(x) = Ae^{kt}$

$$f'(t) = Ake^{kt}$$

c) $f(x) = \sqrt{x} - \sin x$

$$f'(x) = \frac{1}{2}x^{-1/2} - \cos x$$

d) $f(x) = 2^x - \log_2 x$

$$f'(x) = 2^x \ln 2 - \frac{1}{\ln 2}\frac{1}{x}$$

e) $f(x) = 15x^3 - \pi x^2 + \sqrt{2}\,x + C$

$$f'(x) = 45x^2 - 2\pi x + \sqrt{2}$$

2) a) $f(x) = 15x^3 - \pi x^2 + \sqrt{2}x$

$F(x) = \frac{15}{4}x^4 - \frac{\pi}{3}x^3 + \frac{\sqrt{2}}{2}x^2 + C$

b) $f(x) = 4\sin x - \cos x$

$F(x) = -4\cos x - \sin x + C$

c) $f(x) = 2^x - e^x$

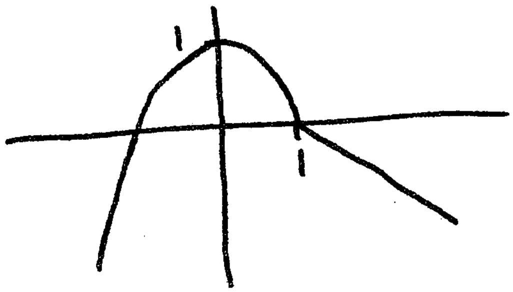
$F(x) = \frac{1}{\ln 2}2^x - e^x + C$

d) $F(x) = kAe^{kt}$

$F(x) = Ae^{kt} + C$

3) $f(x) = \begin{cases} 1-x^2 & x \le 1 \\ 1-x, & x > 1 \end{cases}$

a)



b) $\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^-} 1-x^2 = 1-(1)^2 = 0$

$\lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 1^+} 1-x = 1-(1) = 0$

c) Yes, $\lim\limits_{x\to 1} f(x)$ exists because the right- and left-hand limits exist and are equal.

$$\lim_{x\to 1} f(x) = 0$$

d) No!

$\lim\limits_{x\to 1^-} f'(x) = \lim\limits_{x\to 1^-} (1-x^2)' = \lim\limits_{x\to 1^-} -2x = -2$

not equal ⇒ corner at $x=1$

$\lim\limits_{x\to 1^+} f'(x) = \lim\limits_{x\to 1^+} (1-x)' = \lim\limits_{x\to 1^+} -1 = -1$

4)

$$A = \pi r^2 + 2\pi r h = 10$$

$$\Rightarrow h = \frac{10 - \pi r^2}{2\pi r}$$

$$V = \pi r^2 h = \cancel{\pi} r^{\cancel{2}} \left(\frac{10 - \pi r^2}{2 \cancel{\pi} \cancel{r}} \right)$$

$$= 5r - \frac{\pi r^3}{2}$$

$$V' = 5 - \frac{3\pi r^2}{2} = 0 \Rightarrow 3\pi r^2 = 10$$

$$\Rightarrow r = \pm \sqrt{\tfrac{10}{3\pi}}$$

need positive radius, so

$$\boxed{r = \sqrt{\tfrac{10}{3\pi}}}$$

5) a) $P(t) = 1000 + Ce^{.1t}$

$P'(t) = .1Ce^{.1t}$

$$
\begin{aligned}
.1P - 100 &= .1(1000 + Ce^{.1t}) - 100 \\
&= 100 + .1Ce^{.1t} - 100 \\
&= .1Ce^{.1t}
\end{aligned}
$$

So P satisfies $P' = .1P - 100$

b) Using $P(t) = 1000 + Ce^{.1t}$,

$P(0) = 1000 + C = 2000$

$\Rightarrow C = 1000$

$$
\begin{aligned}
P(10) &= 1000 + 1000e^{.1(10)} \\
&= 1000(1+e) \\
&= \$3718.28
\end{aligned}
$$