HW 11 – 6.2: 2, 3, 6, 23, 25, 27, 32, 40, 44

2. The coefficient, ${}_nC_x$, represents the number of ways to obtain x successes in n trials. Combinations are used because the order of successes is not important.

3. If the binomial probability distribution is roughly bell-shaped, then the Empirical Rule can be used to check for unusual observations. As a rule of thumb, the probability distribution for a binomial random variable will be approximately bell-shaped if $np(1-p) \geq 10$. In a bell-shaped distribution, about 95% of all observations lie within two standard deviations of the mean. That is, about 95% of the observations lie between $\mu - 2\sigma$ and $\mu + 2\sigma$. An observation would be considered unusual if it is more than two standard deviations above or below the mean because this will occur less than 5% of the time.

6. In terms of a binomial experiment, we typically consider "success" to be the outcome we are looking for. For example, a quality control inspector may treat finding a defect as a "success" because he/she is trying to locate defective items.

23. Using $n = 9$ and $p = 0.2$:

$$P(X \leq 3) = P(0) + P(1) + P(2) + P(3)$$
$$= {}_9C_0 \cdot (0.2)^0 (0.8)^9 + {}_9C_1 \cdot (0.2)^1 (0.8)^8 + {}_9C_2 \cdot (0.2)^2 (0.8)^7 + {}_9C_3 \cdot (0.2)^3 (0.8)^6$$
$$\approx 0.134218 + 0.301990 + 0.301990 + 0.176161$$
$$\approx 0.9144$$

25. Using $n = 7$ and $p = 0.5$:

$$P(X > 3) = P(X \geq 4)$$
$$= P(4) + P(5) + P(6) + P(7)$$
$$= {}_7C_4 \cdot (0.5)^4 (0.5)^3 + {}_7C_5 \cdot (0.5)^5 (0.5)^2 + {}_7C_6 \cdot (0.5)^6 (0.5)^1 + {}_7C_7 \cdot (0.5)^7 (0.5)^0$$
$$= 0.2734375 + 0.1640625 + 0.0546875 + 0.0078125$$
$$= 0.5$$

27. Using $n = 12$ and $p = 0.35$:

$$P(X \leq 4) = P(0) + P(1) + P(2) + P(3) + P(4)$$
$$= {}_{12}C_0 \cdot (0.35)^0 (0.65)^{12} + {}_{12}C_1 \cdot (0.35)^1 (0.65)^{11} + {}_{12}C_2 \cdot (0.35)^2 (0.65)^{10}$$
$$+ {}_{12}C_3 \cdot (0.35)^3 (0.65)^9 + {}_{12}C_4 \cdot (0.35)^4 (0.65)^8$$
$$\approx 0.005688 + 0.036753 + 0.108846 + 0.195365 + 0.236692$$
$$\approx 0.5833$$

HWII (2)

32. (a)

Distribution

x	$P(x)$
0	0.1074
1	0.2684
2	0.3020
3	0.2013
4	0.0881
5	0.0264
6	0.0055
7	0.0008
8	0.0001
9	0.0000
10	0.0000

	$x \cdot P(x)$	$(x-\mu_x)^2 \cdot P(x)$
	0.0000	0.4295
	0.2684	0.2684
	0.6040	0.0000
	0.6040	0.2013
	0.3523	0.3523
	0.1321	0.2378
	0.0330	0.0881
	0.0055	0.0197
	0.0006	0.0027
	0.0001	0.0002
	0.0000	0.0000
Σ	2.0000	1.6000

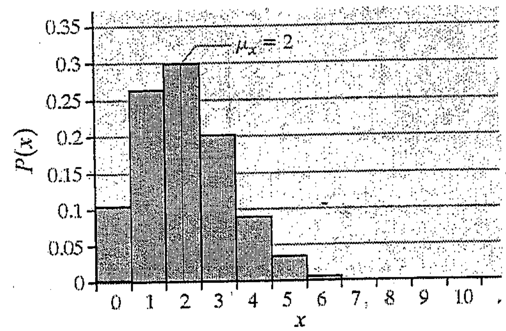
(b) $\mu_X = 2.0$ (from first column in table above and to the right)

$\sigma_X = \sqrt{\sigma_X^2} = \sqrt{1.6000} \approx 1.3$ (from second column in table above and to the right)

(c) $\mu_X = n \cdot p = 10 \cdot (0.2) = 2$ and $\sigma_X = \sqrt{n \cdot p \cdot (1-p)} = \sqrt{2 \cdot (0.8)} \approx 1.3$

(d)

Probability Histogram



The distribution is skewed right.

40. (a) We have $n = 30$, $p = 0.02$, and $x = 3$.

$P(3) = {}_{30}C_3 \cdot (0.02)^3 (0.98)^{27} \approx 0.0188$

There is a 0.0188 probability that in a random sample of 30 patients taking Depakote, exactly 3 will experience weight gain as a side effect.

44. (a) We have $n = 200$ and $p = 0.8$.

$\mu_X = n \cdot p = 200(0.8) = 160$; $\sigma_X = \sqrt{np(1-p)} = \sqrt{160(0.2)} = \sqrt{32} \approx 5.7$

(b) In a random sample of 200 adult smokers, we expect 160 to have started smoking before they were 18 years of age.

(c) Since $np(1-p) = 32 > 10$, we can use the Empirical Rule to check for unusual observations.

180 is above the mean, and we have $\mu_X + 2\sigma_X = 160 + 2(5.7) = 171.4$.

This indicates that 180 is more than two standard deviations above the mean. Therefore, it would be unusual to observer 180 adult smokers out of a random sample of 200 report that they started smoking before they were 18 years of age.

Note: $P(180) = {}_{200}C_{180} \cdot (0.8)^{180} (0.2)^{20} \approx 0.0001$