HW 16 — 8.1: 5, 6, 7, 9, 10, 13, 14, 17, 29, 32

5. The mean of the sampling distribution of $\bar{x}$ is given by $\mu_{\bar{x}} = \mu$ and the standard deviation is given by $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$.

6. To say that the sampling distribution of $\bar{x}$ is normal, we would require that the population be normal. That is, the distribution of X must be normal.

7. four; To see this, note that $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{4n}} = \dfrac{1}{2} \cdot \dfrac{\sigma}{\sqrt{n}}$.

9. The sampling distribution would be exactly normal. The mean and standard deviation would be $\mu_{\bar{x}} = \mu = 30$ and $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{8}{\sqrt{10}} \approx 2.53$.

10. No, because the sample size is large, i.e. $n = 40 \geq 30$. If the distribution of the population is not normal, the sampling distribution is approximately normal with mean $\mu_{\bar{x}} = \mu = 50$ and $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{4}{\sqrt{40}} = \dfrac{2}{\sqrt{10}} = 0.63$.

13. $\mu_{\bar{x}} = \mu = 52$; $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{10}{\sqrt{21}} \approx 2.182$

14. $\mu_{\bar{x}} = \mu = 27$; $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{6}{\sqrt{15}} \approx 1.549$

17. (a) The population must be normally distributed. If this is the case, then the sampling distribution of $\bar{x}$ is exactly normal. The mean and standard deviation of the sampling distribution are $\mu_{\bar{x}} = \mu = 64$ and $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{17}{\sqrt{12}} \approx 4.907$.

(b) $P(\bar{x} < 67.3) = P\left(Z < \dfrac{67.3 - 64}{17/\sqrt{12}}\right) = P(Z < 0.67) = 0.7486$

(c) $P(\bar{x} \geq 65.2) = P\left(Z \geq \dfrac{65.2 - 64}{17/\sqrt{12}}\right) = P(Z \geq 0.24) = 1 - P(Z < 0.24)$

$= 1 - 0.5948 = 0.4052$

HW 16 ②

29. We have a large sample, $n = 60$, so we can use the Central Limit Theorem to say that the sampling distribution of $\overline{x}$ is approximately normal.

$$\mu_{\overline{x}} = \mu = 50;\ \sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}} = \frac{16}{\sqrt{60}}$$

$$P(\overline{x} \le 45) = P\left(Z \le \frac{45-50}{16/\sqrt{60}}\right) = P(Z \le -2.42) = 0.0078$$

This probability is very small. Since it is less than 0.05, it would be unusual for a random sample of 60 college football players from the given population to average 45 or fewer strong blows to the head.

32. **(a)** $\mu = \frac{\sum x}{N} = \frac{854}{6} \approx 142.3$

The population mean running time is 142.3 minutes.

(b) 132, 201; 132, 112; 132, 134, 132, 155; 132, 120; 201, 112; 201, 134; 201, 155; 201, 120; 112, 134; 112, 155; 112, 120; 134, 155, 134, 120; 155, 120

(c) Obtain each sample mean by adding the two running times in a sample and dividing by two.

$$\overline{x} = \frac{132+201}{2} = 166.5 \text{ min};\ \overline{x} = \frac{132+112}{2} = 122 \text{ min};\ \overline{x} = \frac{132+134}{2} = 133 \text{ min}; \text{ etc.}$$

$\overline{x}$	116	122	123	126	127	133	133.5	137.5	143.5	144.5	156.5	160.5	166.5	167.5	178
$P(\overline{x})$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$

(d) $\mu_{\overline{x}} = \left(\frac{1}{15}\right)(116+122+123+\ldots167.5+178) \approx 142.3$ minutes

Notice that this is the same value we obtained in part (a) for the population mean.

(e) $P(127.3 \le \bar{x} \le 157.3) = \frac{6}{15} = 0.4$

(f) For part (b):

132, 201, 112; 132, 201, 134; 132, 201, 155; 132, 201, 120; 132, 112, 134;
132, 112, 155; 132, 112, 120; 132, 134, 155; 132, 134, 120; 132, 155, 120;
201, 112, 134; 201, 112, 155; 201, 112, 120; 201, 134, 155; 201, 134, 120;
201, 155, 120; 112, 134, 155; 112, 134, 120; 112, 155, 120; 134, 155, 120;

For part (c):

$\bar{x}$	Probability	$\bar{x}$	Probability
121.3	1/20	144.3	1/20
122	1/20	148.3	1/20
126	1/20	149	1/20
128.7	1/20	151	1/20
129	1/20	151.7	1/20
133	1/20	155.7	1/20
133.7	1/20	156	1/20
135.7	1/20	158.7	1/20
136.3	1/20	162.7	1/20
140.3	1/20	163.3	1/20

For part (d):

$\mu_{\bar{x}} \approx 142.3$ minutes

For part (e):

$P(127.3 \le \bar{x} \le 157.3) = \frac{14}{20} = 0.7$;

With the larger sample size, the probability of obtaining a sample mean within 15 minutes of the population mean has increased.