

HW 17 - 8.2: 1-10, 13, 16, 18

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1. $0.44; p = \frac{220}{500} = 0.44$

2. sample proportion; $\hat{p} = \frac{x}{n}$

3. False; while it is possible for the sample proportion to have the same value as the population proportion, it will not always have the same value.

4. True; the expected value, or mean, of the sampling distribution of \hat{p} is the population proportion, p .

5. The sampling distribution of \hat{p} is approximately normal if $np(1-p) \geq 10$ (assuming that $n \leq 0.05N$).

6. The standard deviation of \hat{p} decreases as the sample size increases; If the sample size is increased by a factor of 4, the standard deviation of \hat{p} is cut in half.

$$\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

If we increase the sample size by a factor of 4, we have

$$\sqrt{\frac{\hat{p}(1-\hat{p})}{4n}} = \sqrt{\frac{1}{4}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \frac{1}{2} \sigma_{\hat{p}}$$

Thus, increasing the sample size by a factor of 4 will cut the standard deviation of \hat{p} in half.

7. $25,000(0.05) = 1250$

The sample size, $n = 500$, is less than 5% of the population size and

$$np(1-p) = 500(0.4)(0.6) = 120 > 10.$$

The distribution of \hat{p} is approximately normal, with mean $\mu_{\hat{p}} = p = 0.4$ and standard

deviation $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.4(1-0.4)}{500}} \approx 0.022.$

8. $25,000(0.05) = 1250$

The sample size, $n = 300$, is less than 5% of the population size and

$$np(1-p) = 300(0.7)(0.3) = 63 > 10.$$

The distribution of \hat{p} is approximately normal, with mean $\mu_{\hat{p}} = p = 0.7$ and standard

deviation $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.7(1-0.7)}{300}} \approx 0.026.$

9. $25,000(0.05) = 1250$

The sample size, $n = 1000$, is less than 5% of the population size and

$$np(1-p) = 1000(0.103)(0.897) = 92.391 > 10.$$

The distribution of \hat{p} is approximately normal, with mean $\mu_{\hat{p}} = p = 0.103$ and standard

deviation $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.103(1-0.103)}{1000}} \approx 0.010.$

$$10. 25,000(0.05) = 1250$$

The sample size, $n = 1010$, is less than 5% of the population size and $np(1-p) = 1010(0.84)(0.16) = 135.744 > 10$.

The distribution of \hat{p} is approximately normal, with mean $\mu_{\hat{p}} = p = 0.84$ and standard

$$\text{deviation } \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.84(1-0.84)}{1010}} \approx 0.012.$$

HW (2)

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$$13. (a) 1,000,000(0.05) = 50,000$$

The sample size, $n = 1000$, is less than 5% of the population size and $np(1-p) = 1000(0.35)(0.65) = 227.5 > 10$.

The distribution of \hat{p} is approximately normal, with mean $\mu_{\hat{p}} = p = 0.35$ and

$$\text{standard deviation } \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.35(1-0.35)}{1000}} \approx 0.015.$$

$$(b) \hat{p} = \frac{x}{n} = \frac{390}{1000} = 0.39$$

$$P(X \geq 390) = P(\hat{p} \geq 0.39) = P\left(Z \geq \frac{0.39 - 0.35}{\sqrt{0.35(0.65)/1000}}\right) = P(Z \geq 2.65) \\ = 1 - P(Z < 2.65) = 1 - 0.9960 = 0.0040$$

$$(c) \hat{p} = \frac{x}{n} = \frac{320}{1000} = 0.32$$

$$P(X \leq 320) = P(\hat{p} \leq 0.32) = P\left(Z \leq \frac{0.32 - 0.35}{\sqrt{0.35(0.65)/1000}}\right) = P(Z \leq -1.99) = 0.0233$$

$$16. (a) \frac{1136}{0.05} = 22,720$$

Since there are more than 22,720 college students in the US, our sample size, $n = 1136$, is less than 5% of the population size. In addition, $np(1-p) = 1136(0.6)(0.4) = 272.64 > 10$.

The distribution of \hat{p} is approximately normal, with mean $\mu_{\hat{p}} = p = 0.6$ and standard

$$\text{deviation } \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.6(1-0.6)}{1136}} \approx 0.015.$$

$$(b) \hat{p} = \frac{x}{n} = \frac{665}{1136} \approx 0.585$$

$$P(X \leq 665) = P\left(\hat{p} \leq \frac{665}{1136}\right) = P\left(Z \leq \frac{\frac{665}{1136} - 0.6}{\sqrt{0.6(0.4)/1136}}\right) = P(Z \leq -1.01) = 0.1562$$

We have a result that occurs about 16 times in 100. This probability is larger than 0.05 so we would not consider the result to be unusual.

$$(c) \hat{p} = \frac{x}{n} = \frac{725}{1136} \approx 0.638$$

$$P(X \geq 725) = P\left(\hat{p} \geq \frac{725}{1136}\right) = P\left(Z \geq \frac{\frac{725}{1136} - 0.6}{\sqrt{0.6(0.4)/1136}}\right) = P(Z \geq 2.63)$$

$$= 1 - P(Z < 2.63) = 1 - 0.9957 = 0.0043$$

This probability is less than 0.05 so the result would be considered unusual. We obtained a result that would occur about 4 times in 1000.

18. (a) Our sample size, $n = 750$, is less than 5% of the population size and $np(1-p) = 750(0.07)(0.93) = 48.825 > 10$.

The distribution of \hat{p} is approximately normal, with mean $\mu_{\hat{p}} = p = 0.07$ and

$$\text{standard deviation } \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.07(1-0.07)}{750}} \approx 0.009.$$

$$(b) P(\hat{p} > 0.08) = P\left(Z > \frac{0.08 - 0.07}{\sqrt{0.07(0.93)/750}}\right) = P(Z > 1.07)$$

$$= 1 - P(Z \leq 1.07) = 1 - 0.8577 = 0.1423$$

$$(c) \hat{p} = \frac{x}{n} = \frac{40}{750} = \frac{4}{75} \approx 0.053$$

$$P(X \leq 40) = P(\hat{p} \leq 0.053) = P\left(Z \leq \frac{0.053 - 0.07}{\sqrt{0.07(0.93)/750}}\right) = P(Z \leq -1.82) = 0.0344$$

This probability is less than 0.05 so the result would be considered unusual. We obtained a result that would occur about 3 times in 100.