

HW 18 — 9.1: 1-6, 19, 21, 26, 32

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1. The margin of error of a confidence interval of a parameter depends on the level of confidence, the sample size, and the standard deviation of the population.
2. The margin of error increases as the level of confidence increases because, if we want to be more confident that the interval contains the population mean, then we need to make the interval wider.
3. The margin of error decreases as the sample size increases because the Law of Large Numbers states that as the sample size increases the sample mean approaches the value of the population mean.
4. The sample mean is the midpoint of the interval: $\bar{x} = \frac{1}{2}(10+18) = 14$. The margin of error is the distance from the midpoint to the bounds: $E = 18 - 14 = 4$.
5. The mean age of the population is a fixed value (i.e., constant), so it is not probabilistic. The 95% level of confidence refers to confidence in the method by which the interval is obtained, not the specific interval. A better interpretation would be: "We are confident that the interval 21.4 years to 28.8 years, obtained by using our method, is one of the 95% of confidence intervals that contains the mean."
6. Since the margin of error increases as the sample size decreases, rounding down would have the effect of slightly increasing the margin of error beyond its desired value. So, we round up to give a slightly smaller margin of error.

19. (a) For 96% confidence the critical value is $z_{0.02} = 2.05$. Then:

$$\text{Lower bound} = \bar{x} - z_{0.02} \cdot \frac{\sigma}{\sqrt{n}} = 108 - 2.05 \cdot \frac{13}{\sqrt{25}} \approx 108 - 5.33 = 102.67$$

$$\text{Upper bound} = \bar{x} + z_{0.02} \cdot \frac{\sigma}{\sqrt{n}} = 108 + 2.05 \cdot \frac{13}{\sqrt{25}} \approx 108 + 5.33 = 113.33.$$

(b) Lower bound $= \bar{x} - z_{0.02} \cdot \frac{\sigma}{\sqrt{n}} = 108 - 2.05 \cdot \frac{13}{\sqrt{10}} \approx 108 - 8.43 = 99.57$

Upper bound $= \bar{x} + z_{0.02} \cdot \frac{\sigma}{\sqrt{n}} = 108 + 2.05 \cdot \frac{13}{\sqrt{10}} \approx 108 + 8.43 = 116.43.$

Decreasing the sample size increases the margin of error.

(c) For 88% confidence the critical value is $z_{0.06} = 1.555$. Then:

$$\text{Lower bound} = \bar{x} - z_{0.06} \cdot \frac{\sigma}{\sqrt{n}} = 108 - 1.555 \cdot \frac{13}{\sqrt{25}} \approx 108 - 4.04 = 103.96$$

$$\text{Upper bound} = \bar{x} + z_{0.06} \cdot \frac{\sigma}{\sqrt{n}} = 108 + 1.555 \cdot \frac{13}{\sqrt{25}} \approx 108 + 4.04 = 112.04.$$

Decreasing the level of confidence decreases the margin of error.

(d) No. Each sample size is too small to insure the \bar{x} sampling distribution is normal.

(e) The outliers would have increased the mean, shifting the confidence interval to the right. If there are outliers then we should not use this approach to compute a confidence interval.

21. For 95% confidence the critical value is $z_{0.025} = 1.96$. Then:

$$\text{Lower bound} = \bar{x} - z_{0.025} \cdot \frac{\sigma}{\sqrt{n}} = 8.17 - 1.96 \cdot \frac{1.2}{\sqrt{1120}} \approx 8.17 - 0.07 = 8.10 \text{ hours}$$

$$\text{Upper bound} = \bar{x} + z_{0.025} \cdot \frac{\sigma}{\sqrt{n}} = 8.17 + 1.96 \cdot \frac{1.2}{\sqrt{1120}} \approx 8.17 + 0.07 = 8.24 \text{ hours}$$

We are 95% confident that the population mean amount of sleep each night between 8.10 and 8.24 hours.

26. (a) $\bar{x} = \frac{117+95+109+103+111+91+100+99+106}{9} = \frac{2169}{9} \approx 103.4$ minutes

- (b) Yes. All the data values lie within the bounds on the normal probability plot, indicating that the data should come from a population that is normal. The boxplot does not show any outliers.

- (c) For 95% confidence the critical value is $z_{0.025} = 1.96$. Then:

$$\text{Lower bound} = \bar{x} - z_{0.025} \cdot \frac{\sigma}{\sqrt{n}} = 103.4 - 1.96 \cdot \frac{8}{\sqrt{9}} \approx 103.4 - 5.2 = 98.2 \text{ minutes}$$

$$\text{Upper bound} = \bar{x} + z_{0.025} \cdot \frac{\sigma}{\sqrt{n}} = 103.4 + 1.96 \cdot \frac{8}{\sqrt{9}} \approx 103.4 + 5.2 = 108.6 \text{ minutes}$$

We are 95% confident that the population mean flight time for American Airlines flights from Albuquerque to Dallas is between 98.2 and 108.6 minutes.

- (d) For 90% confidence the critical value is $z_{0.05} = 1.645$. Then:

$$\text{Lower bound} = \bar{x} - z_{0.05} \cdot \frac{\sigma}{\sqrt{n}} = 103.4 - 1.645 \cdot \frac{8}{\sqrt{9}} \approx 103.4 - 4.4 = 99.0 \text{ minutes}$$

$$\text{Upper bound} = \bar{x} + z_{0.05} \cdot \frac{\sigma}{\sqrt{n}} = 103.4 + 1.645 \cdot \frac{8}{\sqrt{9}} \approx 103.4 + 4.4 = 107.8 \text{ minutes}$$

We are 95% confident that the population mean flight time for American Airlines flights from Albuquerque to Dallas is between 99.0 and 107.8 minutes.

- (e) When the level of confidence is decreased, the width of the confidence interval is decreased. This result is reasonable because, if we are less confident that the interval will contain the population mean, then the interval does not need to be as wide.

32. (a) $\bar{x} = \frac{\sum x}{35} = \frac{316.67}{35} \approx 9.05$ million shares

- (b) For 90% confidence the critical value is $z_{0.05} = 1.645$. Then:

$$\text{Lower bound} = \bar{x} - z_{0.05} \cdot \frac{\sigma}{\sqrt{n}} = 9.05 - 1.645 \cdot \frac{6.17}{\sqrt{35}} \approx 9.05 - 1.72 = 7.33 \text{ million shares}$$

$$\text{Upper bound} = \bar{x} + z_{0.05} \cdot \frac{\sigma}{\sqrt{n}} = 9.05 + 1.645 \cdot \frac{6.17}{\sqrt{35}} \approx 9.05 + 1.72 = 10.77 \text{ million shares}$$

We are 90% confident that the population mean number of shares of Google stock traded in 2004 is between 7.33 and 10.77 million shares per day.

- (c) The point estimate is $\bar{x} = \frac{\sum x}{35} = \frac{299.49}{35} \approx 8.56$ million shares

$$\text{Lower bound} = \bar{x} - z_{0.05} \cdot \frac{\sigma}{\sqrt{n}} = 8.56 - 1.645 \cdot \frac{6.17}{\sqrt{35}} \approx 8.56 - 1.72 = 6.84 \text{ million shares}$$

$$\text{Upper bound} = \bar{x} + z_{0.05} \cdot \frac{\sigma}{\sqrt{n}} = 8.56 + 1.645 \cdot \frac{6.17}{\sqrt{35}} \approx 8.56 + 1.72 = 10.28 \text{ million shares}$$

We are 90% confident that the population mean number of shares of Google stock traded in 2004 is between 6.84 and 10.28 million shares per day.

- (d) The intervals obtained in parts (a) and (c) are different because the different samples resulted in different sample means. In this case, the sample mean from part (c) is 490,000 shares smaller than the sample mean from part (a).