HW 19 — 9.2: 1, 2, 6, 7, 9, 20, 24, 26 ①

1. We can construct a Z-interval if the sample is random, the population from which the sample is drawn is normal or the sample size is large $(n \geq 30)$, and the population standard deviation, σ, is known. A t-interval should be constructed if the sample is random, the population from which the sample is drawn is normal, but the population standard deviation, σ, is unknown. Neither interval can be constructed if the sample is not random, the population is not normal and the sample size small, or when there are outliers.

2. As the degrees of freedom increase, the t-distribution has less spread because as n increases the values of s becomes closer to the values of σ, by the Law of Large Numbers.

6. Answers will vary. One possibility follows: The degrees of freedom are the number of values that are free to vary after a sample statistic, such as $\bar{x}$, has been calculated.

7. **(a)** From the row with df = 25 and the column headed 0.10, we read $t = 1.316$.
   (b) From the row with df = 30 and the column headed 0.05, we read $t = 1.697$.
   (c) From the row with df = 18 and the column headed 0.01, we read $t = 2.552$. By symmetry, the t-value with an area to the *left* of 0.01 is $t = -2.552$.
   (d) For a 90% confidence interval, we want the t-value with an area in the right tail of 0.05. With df = 20, we read from the tables that $t = 1.725$.

9. **(a)** For 96% confidence, $\alpha/2 = 0.02$. Since $n = 25$, then df = 24. The critical value is $t_{0.02} = 2.172$. Then:

   Lower bound $= \bar{x} - t_{0.02} \cdot \frac{s}{\sqrt{n}} = 108 - 2.172 \cdot \frac{10}{\sqrt{25}} \approx 108 - 4.3 = 103.7$

   Upper bound $= \bar{x} + t_{0.02} \cdot \frac{s}{\sqrt{n}} = 108 + 2.172 \cdot \frac{10}{\sqrt{25}} \approx 108 + 4.3 = 112.3$

   (b) Since $n = 10$, then df = 9. The critical value is $t_{0.02} = 2.398$. Then:

   Lower bound $= \bar{x} - t_{0.02} \cdot \frac{s}{\sqrt{n}} = 108 - 2.398 \cdot \frac{10}{\sqrt{10}} \approx 108 - 7.6 = 100.4$

   Upper bound $= \bar{x} + t_{0.02} \cdot \frac{s}{\sqrt{n}} = 108 + 2.398 \cdot \frac{10}{\sqrt{10}} \approx 108 + 7.6 = 115.6$

   Decreasing the sample size increases the margin of error.

   (c) For 90% confidence, $\alpha/2 = 0.05$. With 24 degrees of freedom, $t_{0.02} = 1.711$. Then:

   Lower bound $= \bar{x} - t_{0.02} \cdot \frac{s}{\sqrt{n}} = 108 - 1.711 \cdot \frac{10}{\sqrt{25}} \approx 108 - 3.4 = 104.6$

   Upper bound $= \bar{x} + t_{0.02} \cdot \frac{s}{\sqrt{n}} = 108 + 1.711 \cdot \frac{10}{\sqrt{25}} \approx 108 + 3.42 = 111.4$

   Decreasing the level of confidence decreases the margin of error.

   (d) No, because in all cases the sample was small $(n < 30)$, so the population must be normally distributed.

HW 19 ②

20. Using technology, we find $\bar{x} \approx 18.71\,\%$ and $s \approx 2.76\,\%$.
For 95% confidence, $\alpha/2 = 0.025$. Since $n = 14$, then df = 13 and $t_{0.025} = 2.160$. Then:

Lower bound $= \bar{x} - t_{0.025} \cdot \dfrac{s}{\sqrt{n}} = 18.71 - 2.160 \cdot \dfrac{2.76}{\sqrt{14}} \approx 18.71 - 1.59 = 17.12\,\%$.

Upper bound $= \bar{x} + t_{0.025} \cdot \dfrac{s}{\sqrt{n}} = 18.71 + 2.160 \cdot \dfrac{2.76}{\sqrt{14}} \approx 18.71 + 1.59 = 20.30\,\%$.

The server is 95% confident that the population mean tip percentage per dinner is between 17.12% and 20.30%.

24. **(a)** Yes. The normal probability plot indicates that the data come from a population that is approximately normal, and the box plot indicates that there are no outliers.

(b) Using technology, $\bar{x} = 9.47$ hours and $s \approx 2.14$ hours.
For 90% confidence, $\alpha/2 = 0.05$. Since $n = 10$, then df = 9 and $t_{0.05} = 1.833$.

Lower bound $= \bar{x} - t_{0.05} \cdot \dfrac{s}{\sqrt{n}} = 9.47 - 1.833 \cdot \dfrac{2.14}{\sqrt{10}} \approx 9.47 - 1.24 = 8.23$ hours.

Upper bound $= \bar{x} + t_{0.05} \cdot \dfrac{s}{\sqrt{n}} = 9.47 + 1.833 \cdot \dfrac{2.14}{\sqrt{10}} \approx 9.47 + 1.24 = 10.71$ hours.

We are 90% confident that the population mean number of hours the battery will last on this player is between 8.23 and 10.71 hours.

(c) The sample size could be increased in order to increase the accuracy of the interval without changing the confidence level.

26. **(a)** Yes. The normal probability plot indicates that the data come from a population that is approximately normal, and the box plot indicates that there are no outliers.

(b) Using technology, $\bar{x} \approx 171.7$ grams and $s \approx 2.0$ grams.
For 95% confidence, $\alpha/2 = 0.025$. Since $n = 12$, then df = 11 and $t_{0.025} = 2.201$.

Lower bound $= \bar{x} - t_{0.025} \cdot \dfrac{s}{\sqrt{n}} = 171.7 - 2.201 \cdot \dfrac{2.0}{\sqrt{12}} \approx 171.7 - 1.3 = 170.4$ grams.

Upper bound $= \bar{x} + t_{0.025} \cdot \dfrac{s}{\sqrt{n}} = 171.7 + 2.201 \cdot \dfrac{2.0}{\sqrt{12}} \approx 171.7 + 1.3 = 173.0$ grams.

We are 95% confident that the population mean disc weight is between 170.4 and 173.0 grams.