

MA 314 May 2008—Homework 4

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Consider the region of \mathbb{R}^3 defined by

$$\frac{x^2}{2^2} + \frac{y^2}{3^2} + \frac{z^2}{4^2} \leq 1$$

that lies in the first quadrant. Call this region A . We want to use Monte Carlo integration to investigate the region A .

- (a) Find the volume of A using Monte Carlo integration. Find the exact volume of this octant of an ellipsoid and comment on the convergence of your Monte Carlo estimate to the true value.
- (b) Recall that the center of mass of a region has coordinates $(\bar{x}, \bar{y}, \bar{z})$ given by

$$\bar{x} = \int_A x \, d\mathbf{x}$$

with the obvious modifications for \bar{y} and \bar{z} . Find the center of mass of A using Monte Carlo integration.

- (c) The moment of inertia of a region is

$$I = \int_A r^2 \, d\mathbf{x}$$

where r is the perpendicular distance from the axis of rotation. Find the moment of inertia of A about the z -axis using Monte Carlo integration.