Equivalence of Measures of Complexity Classes [1]

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Abstract

The resource-bounded measures of complexity classes are shown to be robust with respect to certain changes in the underlying probability measure. Specifically, for any real number $\delta > 0$, any uniformly polynomial-time computable sequence $\vec{\beta} = (\beta_0, \beta_1, \beta_2, \dots)$ of real numbers (biases) $\beta_i \in [\delta, 1-\delta]$, and any complexity class $\mathcal{C}$ (such as P, NP, BPP, P/Poly, PH, PSPACE, etc.) that is closed under positive, polynomial-time, truth-table reductions with queries of at most linear length, it is shown that the following two conditions are equivalent.

(1) $\mathcal{C}$ has p-measure 0 (respectively, measure 0 in E, measure 0 in E_2) relative to the coin-toss probability measure given by the sequence $\vec{\beta}$.

(2) $\mathcal{C}$ has p-measure 0 (respectively, measure 0 in E, measure 0 in E_2) relative to the uniform probability measure.

The proof introduces three techniques that may be useful in other contexts, namely, (i) the transformation of an efficient martingale for one probability measure into an efficient martingale for a "nearby" probability measure; (ii) the construction of a *positive bias reduction*, a truth-table reduction that encodes a positive, efficient, approximate simulation of one bias sequence by another; and (iii) the use of such a reduction to *dilate* an efficient martingale for the simulated probability measure into an efficient martingale for the simulating probability measure.

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