The complexity of parameters for probabilistic and quantum computation

by

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ABSTRACT

In this thesis we study some effects of allowing computational models that use parameters whose own computational complexity has a strong effect on the computational complexity of the languages computable from the model. We show that in the probabilistic and quantum models there are parameter sets that allow one to obtain noncomputable outcomes.

In Chapter 3 we define $\mathrm{BP}_\beta\mathrm{P}$ the BPP class based on a coin with bias β. We then show that if β is BPP-computable then it is the case that $\mathrm{BP}_\beta\mathrm{P} = \mathrm{BPP}$. We also show that each language L in P/CLog is in $\mathrm{BP}_\beta\mathrm{P}$ for some β. Hence there are some β from which we can compute noncomputable languages. We also examine the robustness of the class BPP with respect to small variations from fairness in the coin.

In chapter 4 we consider measures that are based on polynomial-time computable sequences of biased coins in which the biases are bounded away from both zero and one (strongly positive P-sequences). We show that such a sequence $\vec{\beta}$ generates a measure $\mu^{\vec{\beta}}$ equivalent to the uniform measure in the sense that if $\mathcal{C}$ is a class of languages closed under positive, polynomial-time, truth-table reductions with queries of linear length then $\mathcal{C}$ has $\mu^{\vec{\beta}}$-measure zero if and only if it has measure zero relative to the uniform measure μ. The classes P, NP, BPP, P/Poly, PH, and PSPACE are among those to which this result applies. Thus the measures of these much-studied classes are robust with respect to changes of this type in the underlying probability measure.

In chapter 5 we introduce the quantum computation model and the quantum complexity class BQP. We claim that the computational complexity of the amplitudes is

a critical factor in determining the languages computable using the quantum model. Using results from chapter 3 we show that the quantum model can also compute non-computable languages from some amplitude sets. Finally, we determine a restriction on the amplitude set to limit the model to the range of languages implicit in others' typical meaning of the class BQP.